MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

PCL0016 - CALCULUS

(ALL groups)

12 OCTOBER 2019
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This examination paper consists of **3 pages** (including the cover page) with **6 questions only.**

2. **Attempt ALL questions.**

3. Please write all your answers in the Answer Booklet provided. **Show all relevant steps** to obtain the maximum possible marks.

Question 1

(a) Evaluate the limits, if they exist.

(i) $\lim_{x\to e} 4(x-e)^3(\pi-x)$ [2 marks]

(ii) $\lim_{x\to-\infty} \dfrac{\sqrt{x^2+1}}{x-2}$ [3 marks]

(iii) $\lim_{x\to 2} \cos\left(\dfrac{x^2-4}{x-2}\right)$ [5 marks]

(b) Find $\lim_{x\to 3}[f(x)-g(x)]$ if $f(x)=\dfrac{x^2-x-6}{x-3}$ and $g(x)=2$. [5 marks]

(c) For what values of m is the following function continuous at every x?

$$\theta(x)=\begin{cases} mx^2 & , \quad x\le 3 \\ 3x+m & , \quad x>3 \end{cases}$$

[5 marks]

Question 2

(a) Differentiate $f(x)=\dfrac{(\sqrt{x}-1)(2x+1)}{x-4}$. [5 marks]

(b) Given $x\cos y - x^2 = 0$, find dy/dx by using implicit differentiation. [4 marks]

(c) Use a local linear approximation to estimate the value of $\sqrt[5]{3124}$ to 4 d.p.
[Hint: $5^5 = 3125$]

[6 marks]

Question 3

The following polynomial function is given:

$$f(x)=x^3-3x^2$$

(a) Find the y-intercept of the graph of f, if it exists. [1 mark]
(b) Find the interval(s) on which f is increasing or decreasing. [4 marks]
(c) Find the interval(s) on which f is concave up or concave down. [4 marks]
(d) Identify and classify the relative extrema of f. [3 marks]
(e) Sketch the graph of f. [3 marks]

Continued...

Question 4

(a) Evaluate the following integrals:

(i) $\int (1+\cos\theta)^{10}\sin\theta\, d\theta$ [3 marks]

(ii) $\int 2xe^{5x}dx$ [3 marks]

(iii) $\int_0^2 (3x-1)^2 dx$ (via substitution) [3 marks]

(b) Using partial fraction decomposition, show that the rational function

$$g(x)=\frac{1}{(x+1)(x+2)^2}$$

can be written in the form

$$\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{(x+2)^2}.$$ [4 marks]

Hence, determine $\int g(x)dx$. [2 marks]

Question 5

A region R is bounded by $f(x)=x^3+1$ and $g(x)=x/2$ on the interval [0, 2].

(a) Sketch R. [4 marks]

(b) Calculate the area of R. [5 marks]

(c) Using the method of washers, find the volume of the solid generated when R is revolved around the x-axis. [6 marks]

Question 6

(a) Solve $\frac{dy}{dx}-6y^2x=0$ given initial condition $y(1)=\frac{1}{25}$. [6 marks]

(b) Solve the following differential equations:

(i) $y''+3y'-10y=0,\ y(0)=4, y'(0)=-2.$ [7 marks]

(ii) $y''+y=0,\ y(0)=3, y'(0)=-4.$ [7 marks]

End of Paper